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CNC 2005 — Elements de Corrigé

GRUE A CONTAINERS

MP

I- ANALYSE FONCTIONNELLE :

§1]

FT123	Régulateur de vitesse + Frein
FT22	Tendeur de câbles.
FT231	Ensemble moto-Réducteur
FT232	Ensemble moto-Reducteur + frein
FT32	Régulateur de vitesse

II. ETUDE DE FT11 :

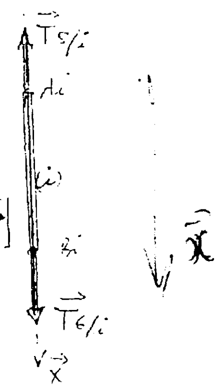
§2] les câbles sont soumis à la Traction.

- Inextensibles → solides
- Infiniment fléxibles ⇒ liai sons Rotules en A_i et B_i

§3] ⊙ on isole (i) :

$$\{\overline{i \to i}\} = \begin{Bmatrix} \vec{0} \\ \vec{0} \end{Bmatrix} \Rightarrow \begin{cases} \vec{R}(\bar{i} \to i) = \vec{0} & (a) \\ \vec{M}_{A_i}(\bar{i} \to i) = \vec{0} & (b) \end{cases}$$

$$\begin{matrix} (a) \Rightarrow \\ (b) \Rightarrow \end{matrix} \left| \begin{matrix} \vec{R}(6 \to i) + \vec{R}(5 \to i) = \vec{0} \\ \vec{R}(6 \to i) \wedge \overrightarrow{B_iA_i} = \vec{0} \end{matrix} \right\} \Rightarrow \boxed{\vec{R}(i \to 6) = -T_i\vec{x}}$$



⊙ on isole (6) :

. Equilibre de (6) ⇒ $\{\overline{6} \to 6\} = \begin{Bmatrix} \vec{0} \\ \vec{0} \end{Bmatrix}$.

. $\boxed{\vec{R}(\bar{6} \to 6) = \vec{0}}$; $\boxed{\vec{M}_{C}(\bar{6} \to 6) = \vec{0}}$

(1) $\boxed{T_1 + T_2 + T_3 + T_4 = Mg}$

$\boxed{(T_3 + T_4 - T_1 - T_2)\frac{L}{2} + Mgc = 0}$ (2)

$\boxed{(T_2 + T_3 - T_1 - T_4)\frac{\ell}{2} - Mgb = 0}$ (3)

⇒ Système hyperstatique d'ordre (1)

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§4] d'après §3] ; on a :

$$\left| \begin{matrix} \sum_{i=1}^{4} T_i - Mg = 0 \\ (-T_1 - T_2 + T_3 + T_4)\frac{L}{2} + Mgc = 0 \\ (-T_1 + T_2 + T_3 - T_4)\frac{\ell}{2} - Mgb = 0 \end{matrix} \right|$$

Avec : $T_i = k\,\Delta L_i$

$$\Rightarrow \boxed{\begin{array}{l} \sum_{i=1}^{4} \Delta L_i - \frac{Mg}{k} = 0 \quad (1) \\ -\Delta L_1 - \Delta L_2 + \Delta L_3 + \Delta L_4 + \frac{2Mgc}{Lk} = 0 \quad (2) \\ -\Delta L_1 + \Delta L_2 + \Delta L_3 - \Delta L_4 - \frac{2Mgb}{\ell k} = 0 \quad (3) \end{array}}$$

§5] ⊙ Torseur ⇒ champs de moments : (champs de petits déplacements)

$$\Rightarrow \boxed{\vec{U}(B_i) = \vec{U}(C) + \vec{\varepsilon}(C) \wedge \overrightarrow{CB_i}}$$

⊙ on a :

$$\Delta L_i\,\vec{x} = \delta\vec{x} + (\alpha\vec{x} + \beta\vec{y} + \gamma\vec{z}) \wedge (Y\vec{y} + Z\vec{z}).$$

$$\left| \Delta L_i\,\vec{x} = \delta\vec{x} + Y\alpha\vec{z} - Z\alpha\vec{y} + Z\beta\vec{x} - Y\gamma\vec{x} \right.$$

$$\Rightarrow \boxed{\Delta L_i = \delta + Z\beta - Y\gamma} \text{ et } |\alpha = 0$$

- Pour $\overrightarrow{CB_1}$: $Y = -\frac{\ell}{2}$; $Z = \frac{L}{2}$
- Pour $\overrightarrow{CB_2}$: $Y = \ell/2$, $Z = \frac{L}{2}$
- Pour $\overrightarrow{CB_3}$: $Y = \ell/2$, $Z = -L/2$
- Pour $\overrightarrow{CB_4}$: $Y = -\ell/2$, $Z = -L/2$

$$\Rightarrow \boxed{\begin{array}{l} \Delta L_1 = \delta + \frac{L}{2}\beta + \frac{\ell}{2}\gamma \\ \Delta L_2 = \delta + \frac{L}{2}\beta - \frac{\ell}{2}\gamma \\ \Delta L_3 = \delta - \frac{L}{2}\beta - \frac{\ell}{2}\gamma \\ \Delta L_4 = \delta - \frac{L}{2}\beta + \frac{\ell}{2}\gamma. \end{array}}$$

Q 6]

$\{D_{(6)}\} = \left\{ \begin{matrix} \vec{\theta}(6) \\ \vec{u}(6) \end{matrix} \right\}_C$;

$m = 20.10^3$ Kg
$k = 2.10^3$ N/mm
$b = 0,3$ m $= 300$ mm
$c = 0,5$ m $= 500$ mm
$L = 3000$ mm
$\ell = 1750$ mm

① ⇒ $\boxed{k\,\delta = \dfrac{Mg}{k} \Rightarrow \delta = 25\ mm}$

② ⇒ $\boxed{L\,\beta = \dfrac{Mg\,c}{L\,k} \Rightarrow \beta = 8.10^{-3}\ rad}$

③ ⇒ $\boxed{\ell\,\gamma = -\dfrac{Mg\,b}{k.\ell} \Rightarrow \gamma = -9{,}8\,10^{-3}\ rad}$

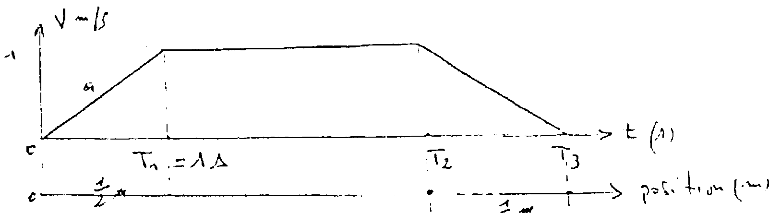
d'où $\boxed{\{D(6)\}_C = \left\{ \begin{matrix} 10^{-3}.(8\vec{y} - 9{,}8\,\vec{z}) \\ 25.10^{-3}\,\vec{x} \end{matrix} \right\}_C}$

⇒

$\Delta L_1 = 46{,}775$ mm ⇒	$T_1 = 93550$ N
$\Delta L_2 = 29{,}625$ mm ⇒	$T_2 = 59250$ N
$\Delta L_3 = 3{,}225$ mm ⇒	$T_3 = 6450$ N
$\Delta L_4 = 20{,}375$ mm ⇒	$T_4 = 40750$ N

III - TRANSFERER : FT2

Q 7]



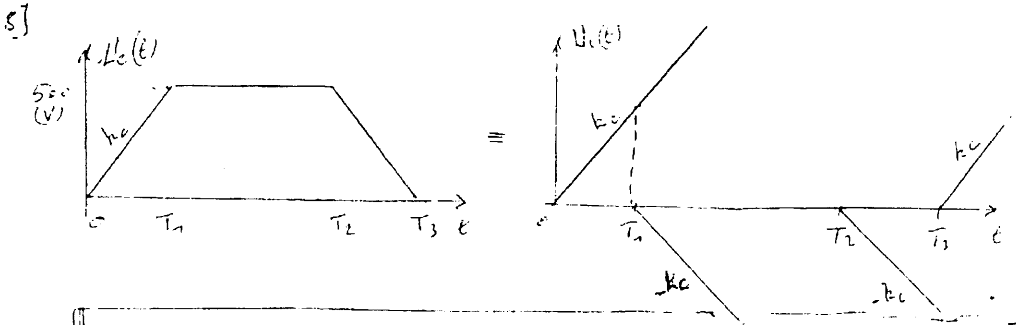
• $\boxed{a = 1\ m/s^2}$

• $T_3 - T_2 = 1s$; $T_1 = 1s$.
• entre 0 et T_1 : $y_{01} = 0{,}5$ m
• " " T_2 et T_3 : $y_{23} = 0{,}5$ m } ⇒ $y_{12} = 59$ m.
• entre T_1 et T_2 : M^{vt} uniforme ⇒ $y(t) - y_1 = v_{max}\,t$

$t = \dfrac{y_2 - y_1}{V_{max}} = \dfrac{59}{1} = \boxed{59\,s}$

⇒ $T_1 = 1s$; $\boxed{T_2 = 60\,s}$, $\boxed{T_3 = 61\,s}$

Q 8]



• $\boxed{U_c(t) = kc.\left[t.u(t) - (t - T_1).u(t - T_1) - (t - T_2).u(t - T_2) + (t - T_3).u(t - T_3)\right]}$

$k_c = 500$

⇒ $\boxed{U_c(p) = \dfrac{k_c}{p^2}.\left[1 - e^{-T_1 p} - e^{-T_2 p} + e^{-T_3 p}\right]}$

Q 9] Ecart $\varepsilon_s = 0$ car présence d'1 Intégration dans la Fonction de Transfert en B.O

B - TRANSFERT du CONTAINER :

Q 10] ⊙ câbles de même longueur, flexibles et restent // :
⇒ M^{vt} 6/5 : Translation circulaire.

⊙ $\vec{\Omega}6/0 = \underbrace{\vec{\Omega}6/5}_{=\vec{0}} + \underbrace{\vec{\Omega}5/0}_{=\vec{0}} = \vec{0}$; (5/0 est glissière).

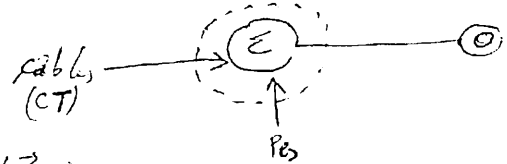
(5)

Q1.1] • $\vec{V}_{G/0} = \frac{d}{dt}\vec{OG}\Big]_0 = \Big[\frac{d}{dt}\vec{OP} + \cancel{\vec{PA}} + \vec{AB} + \cancel{\vec{BG}}\Big]_0$

$$\boxed{\vec{V}_{G/0} = \dot{y}\,\vec{y} + l\,\dot{\theta}\,\vec{y}_1}$$

• $\vec{\Gamma}_{G/0} = \frac{d}{dt}\vec{V}_{G/0}\Big]_0 \Rightarrow \boxed{\vec{\Gamma}_{G/0} = \ddot{y}\,\vec{y} + l\,[\ddot{\theta}\,\vec{y}_1 - \dot{\theta}^2\,\vec{x}_1]}$

Q1.2] $\Sigma = \{5 + 6 + \text{câbles}\}$.

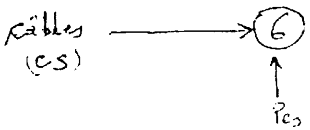


• TRD à $\Sigma / \vec{y} \Rightarrow$

$\vec{y}\cdot\vec{R}(PS \to \Sigma) + \vec{y}\cdot\vec{R}(CT \to \Sigma) + \cancel{\vec{y}\cdot\vec{R}(0 \to 5)} = m\ddot{y} + M\vec{y}\cdot\vec{\Gamma}_{G/0}$

$$\Rightarrow \boxed{\Delta T = (M+m)\,\ddot{y} + M\,l\,\ddot{\theta}\cos\theta - M\,l\,\dot{\theta}^2\sin\theta}$$

Q1.3] $\{6\}$.



• TRD à $6 / \vec{y}_1 \Rightarrow$.

$\cancel{\vec{y}_1\cdot\vec{R}(CS \to 6)} + \vec{y}_1\cdot\vec{R}(PS \to 6) = \vec{y}_1\cdot M\,\vec{\Gamma}_{G/0}$

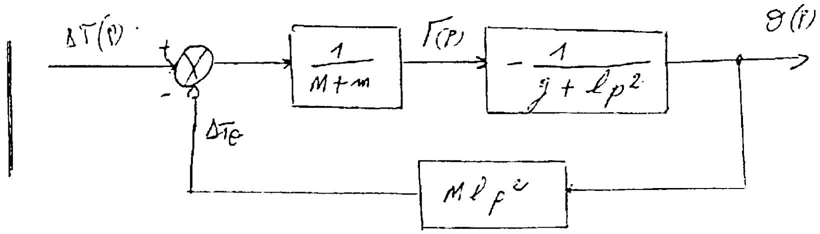
$$\Rightarrow \boxed{-Mg\sin\theta = M\ddot{y}\cos\theta + M\,l\,\ddot{\theta}}$$

Q1.4] $\theta \simeq 0 \Rightarrow \underline{\cos\theta = 1}$; $\underline{\sin\theta = \theta}$; $\underline{\theta\cdot\dot{\theta}^2 \simeq 0}$.

$$\Rightarrow \left|\begin{array}{l} (M+m)\ddot{y} + M\,l\,\ddot{\theta} = \Delta T \\ \theta + \frac{\ddot{y}}{g} + \frac{l\ddot{\theta}}{g} = 0 \end{array}\right.$$

(6)

Q1.5] $\left|\begin{array}{l} \mathcal{L}(1) \Rightarrow (M+m)\Gamma(p) + M\,l\,p^2\,\theta(p) = \Delta T(p). \\ \mathcal{L}(2) \Rightarrow \theta(p)\left[1 + \frac{l}{g}p^2\right] + \frac{\Gamma(p)}{g} = 0 \end{array}\right.$



Q1.6] $\boxed{\frac{\theta(p)}{\Delta T(p)} = -\frac{\frac{1}{g(M+m)}}{1 + \frac{m\,l}{g(M+m)}\cdot p^2}}$; Amortissement $\underline{\text{nul}}$

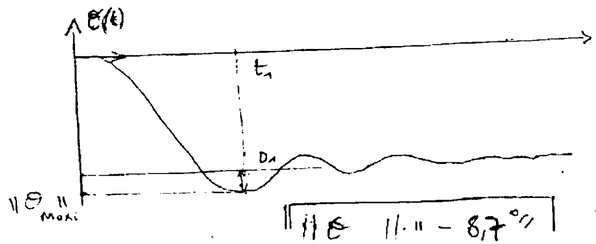
Q1.7) $\frac{\ddot{y}}{g} = \frac{2\xi\dot{\theta}}{\omega_0} \Rightarrow \boxed{\ddot{y} = \frac{2\xi g\dot{\theta}}{\omega_0}}$

Q1.8) $\ddot{y}(t) = \Gamma_0 + \gamma(t)$, $\Gamma_0 = u(t)$; $\gamma(t) = \frac{2\xi}{\omega_0}\dot{\theta}g$

(2) devient : $\theta(t) + \frac{\Gamma_0 + \gamma(t)}{g} + \frac{l}{g}\ddot{\theta} = 0$

$\theta(t) + \frac{\Gamma_0}{g} + \frac{2\xi\dot{\theta}}{\omega_0} + \frac{l}{g}\ddot{\theta} = 0$

$\boxed{\theta(t) + \frac{2\xi}{\omega_0}\dot{\theta} + \frac{l}{g}\ddot{\theta} = -\frac{\Gamma_0}{g}}$, 2ème ordre, second membre cte ; $\xi = 0{,}2$



$\left|\begin{array}{l} D_1 = -\frac{1}{g}\cdot e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \\ D_1 = -3\,\% \end{array}\right.$

$\theta(\infty) = -0{,}1$ rad

$\boxed{\|\theta\|_{\text{maxi}} = -8{,}7\,\%}$

(7)

Q19] $\Gamma_0 = 0 \Rightarrow \theta(\infty) = 0$.

Q20] Identification : $\frac{\Delta T(p)}{Uc(p)}$.

20a) d'après figs doc 4 : l'ordre est ①.
car pente à l'origine nulle.

20b) • Gain statique (K) : $\Delta T(\infty) = K \cdot 500 = 100$

$\Rightarrow \boxed{K = 0,2}$

• Temps de réponse : $\boxed{t_{r\,5\%} \approx 1,2\,s}$

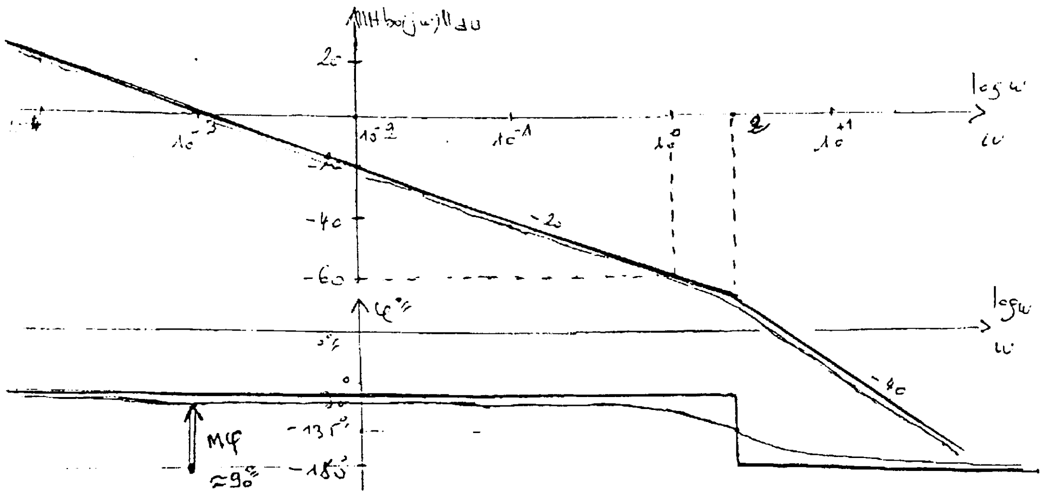
20c) Cte de temps :

Si 1er ordre alors $t_{r\,5\%} = 3 \cdot \tau = 1,2\,s$.

$\Rightarrow \boxed{\tau = \frac{1,2}{3} = 0,4\,s}$

$\Rightarrow \boxed{\frac{\Delta T(p)}{Uc(p)} = \frac{0,2}{1 + 0,4p}}$

Q21] ona : $\frac{V(p)}{Uc(p)} = \frac{10^{-3}}{p(1+0,5p)} = Hbo(p)$; $(C(p)) = 1$



(8)

Q22] $Hbo_c(p) = \frac{10^{-3} \cdot C}{p(1+0,5p)} \Rightarrow Hbf(p) = \frac{10^{-3} C}{10^{-3} C + p + 0,5 p^2}$

$$\boxed{Hbf(p) = \frac{1}{1 + \frac{10^3}{C} p + \frac{0,5 \cdot 10^3}{C} p^2}}$$

$y(t)$ plus rapide $\Rightarrow z = 0,7$.

or : $\omega_c = \sqrt{\frac{C}{500}}$, $\frac{2z}{\omega_c} = \frac{1000}{C}$

$\Rightarrow z = \frac{1000 \cdot \omega_c}{2C} = 10^3 \cdot \frac{1}{2 \cdot \sqrt{500} \cdot \sqrt{C}}$

$z = \frac{10^3}{20 \sqrt{5 \cdot C}}$

$\Rightarrow C = \frac{2500}{(0,7)^2 \cdot 5} \approx 1020,5$

$\boxed{C = 1020,5}$

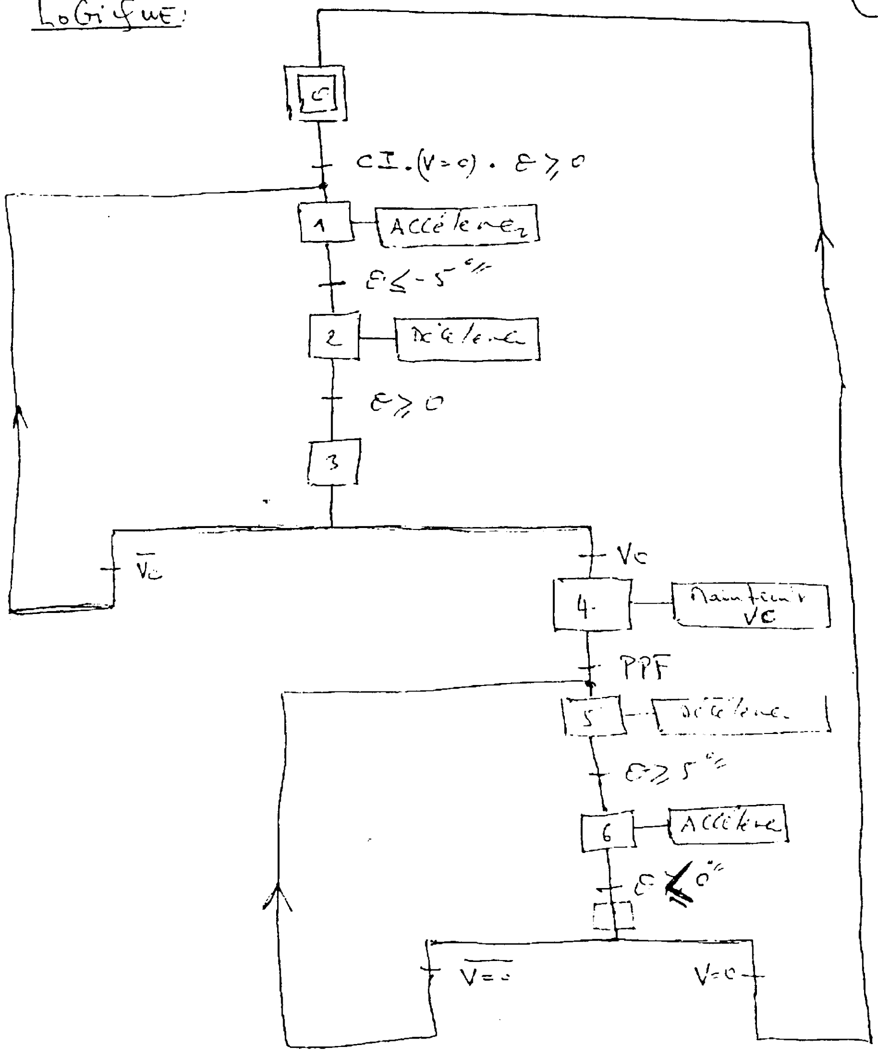
Q23]

23a)
I : Position y (m).
II : position θ (rad).
III : Vitesse V (m/s)

23b) • Avant l'arrêt consigne permanent : $V(\infty) = 0,8\,m/s$.

23c) Amplitude Maxi $\approx 0,04\,rad \approx 3°$

Q24] LoGique:



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